Національний технічний університет України «КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ імені ІГОРЯ СІКОРСЬКОГО»

MMCA

Department of Mathematical Methods of System Analysis

Applied Nonlinear Analysis

Work program of the discipline (Syllabus)

Details of the discipline	
Level of higher education	***Third (educational and scientific)***
Field of knowledge	*12 Information Technology*
Speciality	*124 System Analysis*
Educational program	*System Analysis*
Discipline status	*Normative*
Form of study	*full-time (daytime)/full-time (evening)/part-time/remote/mixed*
Year of preparation, semester	*2nd year, autumn/spring semester*
Scope of discipline	*5 ECTS credits*
Semester control / control measures	*Exam*
Timetable	
Language of instruction	*Ukrainian/English*
Information about Course Leader / Instructors	Lecturer: *Doctor of Physical and Mathematical Sciences, Professor, Corresponding Member of the National Academy of Sciences of Ukraine Kasyanov Pavlo Olehovych, kasyanov.pavlo@lll.kpi.ua https://www.facebook.com/pkasyanov https://www.linkedin.com/in/pavlokasyanov/ https://www.researchgate.net/profile/Pavlo_Kasyanov* Practical / Seminar: *Doctor of Physical and Mathematical Sciences, Professor, Corresponding Member of the National Academy of Sciences of Ukraine Kasyanov Pavlo Olehovych*, kasyanov.pavlo@lll.kpi.ua https://www.facebook.com/pkasyanov https://www.linkedin.com/in/pavlokasyanov/ https://www.researchgate.net/profile/Pavlo_Kasyanov
Course Placement	https://do.ipo.kpi.ua/course/view.php?id=2119

The program of the discipline

1. Description of the discipline, its purpose, subject of study and learning outcomes

The purpose of the credit module is to form the following competencies in graduate students:

general *- GC 2 Ability to search, process and analyze information from various sources; GC 3 Ability to communicate in a foreign language to the extent sufficient to present and discuss the results of their scientific work in oral and written form, as well as to fully understand foreign scientific texts in the specialty; GC 4 Ability to independently conduct research activities, including analysis of problems, setting goals and objectives, selection of means and methods of research, as well as assessment of its quality; GC 7 Ability for continuous self-development and self-improvement;*

professional *– FC 2 Ability to comply with moral and ethical rules of conduct, research ethics, characteristics for participants in the academic environment, as well as the rules of academic integrity in scientific research;*

Acquisition of programmatic learning outcomes:

PRN 7 Know the methods of applied nonlinear analysis and use them in the development of mathematical models of nonlinear nonstationary processes; PRN 17 Read and understand foreign texts in the specialty; freely present and discuss with specialists and non-specialists the results of research, scientific and applied problems of the industry in the state and foreign languages, competently reflect the results of research in scientific publications in leading international scientific journals; PRN 18 Adhere to the rules of academic integrity; know and adhere to the basic principles of academic integrity in scientific and educational (pedagogical) activities.

Subject of study.

Complex Nonlinear Systems, Applied Problems of Nonlinear Analysis of Various Nature.

The main tasks of the credit module.

According to the requirements of the program of the discipline, students after mastering the credit module must demonstrate the following learning outcomes:

***Knowledge**:*

basic concepts, approaches and methods of nonlinear and multivalued analysis, basic methods of qualitative analysis of nonlinear systems of partial differential equations;

***Skills**:*

use modern mathematical apparatus and the basics of nonlinear and multivalued analysis; apply basic theoretical and practical methods of qualitative analysis of nonlinear systems of partial differential equations with multivalued or discontinuous right-hand side;

***Experience**:*

application of the methodology of nonlinear and multivalued analysis to specific problems that are mathematical models of processes of different nature

2. Prerequisites and post-requisites of the discipline (place in the structural and logical scheme of training in the relevant educational program)

Basic level of English, higher mathematics, programming.

3. The content of the discipline

Credit module 1.

Chapter 1. Elements of Applied Nonlinear Analysis

1.1. Elements of Applied Functional Analysis

1.1.1. Metric spaces. Basic concepts and theorems.

1.1.2. Normalized spaces. Approximation problems in normalized spaces.

1.1.3. Functionality. Conjugate spaces. Differentiation in normalized spaces.

1.1.4. Problems of calculus of variations.

1.2. Variational inequalities in finite-dimensional space.

1.2.1. Fixed points. Properties of projection on a convex set.

1.2.2. The first theorem on variational inequalities. Variational inequalities. Some problems that lead to variational inequalities.

1.3. Variational inequalities in Hilbert space.

1.3.1. Bilinear forms. Existence of solutions. Slicing.

1.3.2. Sobolev spaces and boundary value problems. The weak principle of maximum.

1.3.3. Obstacle problem. Initial properties. An obstacle problem in a one-dimensional case.

1.4. Properties of solutions of variational inequalities. Regularity problems.

1.4.1. Abstract existence theorem.

1.4.2. Non-coercive operators. Semilinear equations.
1.4.3. Quasilinear operators.
1.4.4. Penalty method. Dirichlet integral.

4. Training Materials & Resources

All the necessary materials are contained on the Sikorsky platform (Moodle KPI)

https://do.ipo.kpi.ua/course/view.php?id=2119

4.1. Basic

1. Elements of nonlinear analysis. Part I: Introduction to Applied Functional Analysis / O.V. Kapustyan, N.V. Gorban, L.S. Paliychuk, I.D. Fartushnyi, O.V. Khomenko. – K.: NTUU "KPI", 2015. – 106 p.

2. AubinJ.-P. Set-valued analysis / J.-P. Aubin, H. Frankowska. – Boston: Birkhauser, 1990. – 461 p.

4.2. Auxiliary

1. Bellman R. Dynamic Programming / Bellman R. – Princeton, New Jersey: Princeton Univ. Press, 1957.

2. Berge C. Topological Spaces / Berge C. – New York: Macmillan, 1963. – 270 p.

3. Bertsekas D.P. Infinite-time reachability of state-space regions by using feedback control / D.P. Bertsekas // IEEE Trans. Automatic Controll, AC-17. - 1972. – P. 604 – 613.

4. Bertsekas D.P.Dynamic Programming and Stochastic Control / D.P. Bertsekas. – New York: Academic Press, 1976.

5. Bertsekas D.P. Stochastic Optimal Control: The Discrete-Time Case / D.P. Bertsekas, S.E. Shreve. – Belmont, MA: Athena Scientific, 1996. – 330 p.

6. Blackwell D. On stationary policies / D. Blackwell // J. Roy. Statist. Soc. – 1970. – Vol. 133A. – P. 33 – 37.

7. Bogachev V. Measure Theory. Volume II / Bogachev V. – Berlin: Springer-Verlag, 2007. – 586 p.

8. Chen X. Coordinating Inventory Control and Pricing Strategies with Random Demand and Fixed Ordering Cost: The Infinite Horizon Case / Xin Chen, David Simchi-Levi // Mathematics of Operation Research. – 2004. – Vol. 29, No. 3. – P. 698 – 723. 146

9. Clarke F.H. Optimization and Nonsmooth Analysis / Clarke F.H. – New York: John Wiley & Sons, Inc., 1983. – 308 p.

10. Derman C. Finite State Markovian Decision Processes / Derman C. – New York: Academic Press, 1970.

11. Feinberg E.A. Optimality Inequalities for Average Cost Markov Decision Processes and the Stochastic Cash Balance Problem / Eugene Feinberg, Mark Lewis // Mathematics of Operation Research. – 2007. – Vol. 32, No. 4. – P. 769 – 783.

12. Halmos P.R. Measure Theory / Halmos P.R. – Princeton, New Jersey: Van Nostrand-Reinhold, 1950.

13. Hernandez-Lerma O. Average optimality in dynamic programming on Borel spaces – Unbounded costs and controls / O. Hernandez-Lerma // Systems & Control Letters. – 1991. – Vol. 27. – P. 237 – 242.

14. Hernandez-Lerma O. Discrete-Time Markov Control Processes: Basic Optimality Criteria / O. Hernandez-Lerma, J.B. Lassere. – New York: Springer, 1996. – 216 p.

15. Hernandez-LermaO. Monotone approximations for convex stochastic control problems / Onesimo Hernandez-Lerma, Wolfgang Runggaldier // J. Math. Syst., Estimation, and Control. – 1994. – Vol. 4. – P. 99 – 140.

16. Kushner H. Introduction to Stochastic Control / Kushner H. – New York: Holt, 1971.

17. Ornstein D. On the existence of stationary optimal strategies / D.Ornstein // Proc. Amer. Math. Soc. – 1969. – Vol. 20. – P. 563 – 569.

18. Ortega J.M. Iterative Solutions of Nonlinear Equations in Several Variables / J.M. Ortega, W.C. Rheinboldt. – New York: Academic Press, 1970.

19. Rieder U. Measurable selection theorems for optimization problems / U. Rieder // Manuscripta Math. – 1978. – Vol. 24. – P. 115 – 131.

20. Schal M. Average optimality in dynamic programming with general state space / M. Schal // Math. Oper. Res. – 1993. – Vol. 18, No. 1. – P. 163 – 172.

21. Schal M. A Selection Theorem for Optimization Problems / M. Schal // Arch. Math. – 1974. – Vol. 25. – P. 219 – 224.

22. Schochetman I. E. Existence and Discovery of Average Optimal Solutions in Deterministic Infinite Horizon Optimization / Irwin Schochetman, Robert Smith // Mathematics of Operations Research. – 1998. – Vol. 23, No. 2. – P. 416 – 432.

23. Serfozo R. Convergence of Lebesgue integrals with varying measures / R. Serfozo // The Indian Journal of Statistics (Series A). – 1982. – Vol. 44. – P. 380 – 402.

24. Strauch R.E. Negative dynamic programming / R.E. Strauch // Ann. Math. Statist. – 1966. – Vol. 37. – P.871 – 890.

25. Wachs A.O. Average Optimality in Nonhomogeneous Infinite Horizon Markov Decision Processes / A.O. Wachs, I.E. Schochetman, R.L. Smith // Mathematics of Operations Research. – 2011. – Vol. 36, No. 1. – P. 147 – 164.

26. Zgurovsky M.Z. Evolution Inclusions and Variation Inequalities for Earth Data Processing I / Zgurovsky M.Z., Mel'nik V.S., Kasyanov P.O. – Berlin: Springer, 2011. – 247 p.

Educational content

5. Methods of mastering the discipline (educational component)

5.1. Lectures

Salary No.	*Title of the topic of the lecture and a list of the main questions (list of didactic aids, references to literature and tasks for the SRS)*
1	*Metric spaces. Basic concepts and theorems [1]. Tasks for independent work: work through the theoretical material according to the synopsis*
2	*Normalized spaces. Approximation problems in normalized spaces [1]. Tasks for independent work: work through the theoretical material according to the synopsis*
3	*Functionality. Conjugate spaces. Differentiation in normalized spaces [1]. Tasks for independent work: work through the theoretical material according to the synopsis*

4	*Problems of calculus of variations [1]. Tasks for independent work: work through the theoretical material according to the synopsis*
5	*Fixed points. Properties of projection on a convex set [2,3]. Tasks for independent work: work through the theoretical material according to the synopsis*
6	*First theorem on variational inequalities. Variational inequalities. Some problems that lead to variational inequalities [3,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
7	*Bilinear forms. Existence of solutions. Cutting [2,3]. Tasks for independent work: work through the theoretical material according to the synopsis*
8	*Sobolev Spaces and Boundary Value Problems. The weak principle of the maximum [3,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
9	*Obstacle problem. Initial properties. An obstacle problem in a one-dimensional case [3,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
10	*Abstract existence theorem [2,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
11	*Non-coercive operators. Semilinear equations [2,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
12	*Quasilinear operators [2,8]. Tasks for independent work: work through the theoretical material according to the synopsis*
13	*Penalty method. Dirichlet integral [2,3]. Tasks for independent work: work through the theoretical material according to the synopsis*

6. Independent work of a student/graduate student

Students' independent work consists of processing materials and completing tasks on the Sikorsky distance learning platform (Moodle KPI) https://do.ipo.kpi.ua/course/view.php?id=2119 *preparation for the exam.*

Policy & Control

7. Academic discipline policy (educational component)

Proper execution of all tasks on the Sikorsky remote platform (Moodle KPI) is required

https://do.ipo.kpi.ua/course/view.php?id=2119 *according to the requirements and individual strategy, which is determined by the graduate student independently or, if necessary, under the scientific guidance of the teacher / supervisor.*

8. Types of control and rating system for assessing learning outcomes (CRO)

Current control: each student determines the strategy for completing tasks (independently or, if necessary, under the scientific guidance of the teacher / supervisor), aiming to receive 100 points at the end of the semester.

Types of control:

1) two answers (each student on average) in the classroom (provided that an average of 8 students are interviewed in one lesson, with a maximum group size of 30 people);

2) performance of one test (remotely – tests and tasks).

RATING POINTS SYSTEM

1. Classes
The maximum number of points in all classes is 20 points X 2 = 40 points.
Evaluation criteria:
0-8 points – the problem is not solved, while the student has certain theoretical information about the topic of the practical lesson;
9-14 points – the problem is not fully solved or the solution contains gross technical shortcomings, while the student is fluent in theoretical information about the topic of the practical lesson;
15-20 points – the problem is solved as a whole, while the student is fluent in theoretical information about the topic of the practical lesson.

2. Modular control.
The maximum number of points for a test (project) is 60 points.
Evaluation criteria:
0-20 points – the problem as a whole has not been solved or the solution contains gross technical shortcomings, there is no answer to the theoretical question;
21-50 points – the problem is solved as a whole, the theoretical issue is disclosed;
51-60 points – the problem is solved, the answer to the theoretical question is exhaustive.
Penalty and incentive points for:
- performance of tasks to improve didactic materials in disciplines is given from 15 to 30 incentive points.
According to the results of academic work in the first 7 weeks, the "ideal student" should score 20 points. At the first attestation (8th week), a student receives an "enrolled" if his current rating is not less than 10 points. According to the results of 13 weeks of study, the "ideal student" should score 40 points. At the second attestation (14th week), the student receives "passed" if his current rating is not less than 20 points. The maximum amount of points is 100. A prerequisite for admission to the test is a positive mark on the test. To receive credit from the credit module "automatically", you need to have a rating of at least 60 points, as well as a credited test (more than 30 points). Students who have a rating of less than 60 points at the end of the semester, as well as those who want to improve their grade in the ECTS system, complete the test work. At the same time, the points for the test are added to the points for the test work, and this rating score is final. The control task of this work consists of two questions of the work program from the list provided in the methodological recommendations for mastering the credit module. An additional question on the topics of practical classes is given to students who did not take part in the work of a particular practical lesson. An unsatisfactory answer to an additional question lowers the overall score by 4 points.
Each question is scored out of 20 points according to the grading system:
– "excellent", full answer (at least 90% of the required information) – 20... 18 points;
– "good", a fairly complete answer (at least 75% of the required information, or minor inaccuracies) – 17... 14 points;
– "satisfactory", incomplete answer (at least 60% of the required information and some errors) – 13... 11 points;
– "unsatisfactory", unsatisfactory answer – 0 points.
Sum of points: for each of the two questions of the test and the test, it is transferred to the credit grade according to the table.
Points Score
100… 95 Excellent
94… 85 Very Good
84… 75 Good
74… 65 Satisfactory
64… 60 Enough
Less than 60 Unsatisfactory
R&D Not Credited Not Allowed

9. Additional information on the discipline (educational component)

All the necessary materials are contained on the Sikorsky learning platform (Moodle KPI)

https://do.ipo.kpi.ua/course/view.php?id=2119

Work program of the discipline (syllabus):

Compiled by Director of IASA, Doctor of Physical and Mathematical Sciences, Professor, Kasyanov Pavel Olegovich

Academician of the National Academy of Sciences of Ukraine, Doctor of Technical Sciences, Prof. Mikhail Zakharovich Zgurovsky

Approved by the Department of Mathematical Methods of System Analysis (Minutes No. 13 dated 05.06.2024)

Approved by the Methodological Commission of the Faculty (Minutes No. 10 dated 24.06.2024)